SOME PROPERTIES OF A CYLINDRICAL PLASMA CAPACITOR

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This paper examines the transients that occur in a cylindrical plasma capacitor in a static magnetic field H_0 when a constant electric field E_0 perpendicular to the magnetic is suddenly applied to the plasma layer. The time characteristics of the transients are determined by the dynamics of the plasma polarization in the external electric and magnetic fields [1, 2]. Just as in [1, 2], we shall consider the electrostatic polarization of the plasma when flow of the steady current through the plasma is stopped.

1. We shall consider two coaxial cylinders with inside radius r_1 and outside radius R_1 (Fig. 1) unbounded along the z-axis. In the space between the cylinders is a cylindrical layer of homogeneous two-component partially ionized plasma with inside and outside radii r_0 and R, respectively. The magnetic field H_0 is directed along the z-axis. The entire system is symmetric relative to the axis of the cylinders. The space between the cylinders is divided into three regions: 1 ($r_1 \leq r \leq r_0$), 2 ($r_0 \leq r \leq R$), and 3 ($R \leq r \leq R_1$). Charges are absent in regions 1 and 3.

At time t = 0, a potential difference U_0 is applied to the cylinders such that the inside cylinder has the potential $U_0/2$ and the outside $-U_0/2$. The electric field at t = 0 is the same in all three regions and is $E_0 = U_0/R_1 - r_1$ (E_0 has only a radial component), since at time t = = 0 charge separation has not yet occurred in the plasma, i.e., it is assumed that the voltage on the capacitor plates is established much sooner than substantial changes occur in the plasma.

In the presence of external electromagnetic fields, Lorentz forces act on the electrons and ions, owing to which the electrons and ions in the plasma move, colliding with the neutral atoms of the gas with the frequencies ν_e and ν_p, respectively. The motion of charged particles in the plasma changes the electron and ion concentrations $n_e(r, t)$ and $n_p(r, t)$ from the common initial concentration level $N_e = N_p = n_0 =$ = const.

The heterogeneous concentration of electrons and ions in the plasma causes diffusion currents. And in addition, since $n_e(r, t) \neq n_p(r, t)$, a space-charge field is produced, which, in turn, affects the motion of the charged particles (the electron and ion temperatures are considered constant and equal T_e and T_p, respectively).

2. Using the expressions for $v_{er}(r, t)$ and $v_{pr}(r, t)$ that were obtained in [2] from the continuity equations, it is easy to find $n_e(r, t)$ and $n_p(r, t)$ with allowance for the obvious initial condition

$$n_e(r, 0) = n_p(r, 0) = 0,$$

$$\begin{aligned} n_e(r, t) = &- n_0 \sum_j \lambda_j M_j(r) \Big\{ A_e \Big(1 - \exp\frac{-\nu_e t}{2} \cos\delta_e t\Big) - \\ &- (B_e + b_e t) \exp\frac{-\nu_e t}{2} \sin\delta_e t + \\ &+ C_e \Big(1 - \exp\frac{-\nu_p t}{2} \cos\delta_p t\Big) - \\ &- E_e \exp\frac{-\nu_p t}{2} \sin\delta_p t - Gt \exp\frac{-\nu_e t}{2} \cos\delta_e t + \\ &+ \frac{S_e}{\nu_e}(1 - \exp - \nu_e t)\Big\}, \end{aligned}$$

$$\begin{aligned} n_p(r, t) = n_0 \sum_j \gamma_j L_j(r) \Big\{ A_p \Big(1 - \exp\frac{-\nu_p t}{2} \cos\delta_p t\Big) - \\ - (B_p + b_p t) \exp\frac{-\nu_p t}{2} \sin\delta_p t - \\ - G_p t \exp\frac{-\nu_p t}{2} \cos\delta_p t + \frac{S_p}{\nu_p}(1 - \exp - \nu_p t)\Big\}, \end{aligned} \quad (2.1)$$

where

$$M_j(r) = \frac{I_0(\lambda_j r)}{I_1(\lambda_j r_0)} - \frac{y_0(\lambda_j r)}{y_1(\lambda_j r_0)},$$

$$L_j(r) = \frac{I_0(\gamma_j r)}{I_1(\gamma_j r_0)} - \frac{I_0(\gamma_j r)}{I_1(\gamma_j r_0)},$$

$$A_{ep} \frac{1}{{}^1\!/_4\nu_{ep}^2 + \delta_{ep}^2}\Big\{P_{ep}\delta_{ep} + Q_{ep}\frac{\nu_{ep}}{2} + + \frac{p_{ep}\nu_{ep}\delta_{ep} + q_{ep}(\nu_{ep/4}^2 - \delta_{ep}^2)}{{}^1\!/_4\nu_{ep}^2 + \delta_{ep}^2}\Big\},$$

$$B_{ep} = \frac{1}{{}^1\!/_4\nu_{ep}^2 + \delta_{ep}^2}\Big\{P_{ep}\frac{\nu_{ep}}{2} - Q_{ep}\delta_{ep} - - \frac{q_{ep}\nu_{ep}\delta_{ep} - p_{ep}({}^1\!/_4\nu_{ep}^2 - \delta_{ep}^2)}{{}^1\!/_4\nu_{ep}^2 + \delta_{ep}^2}\Big\},$$

$$C_e = \frac{R_e\delta_p}{{}^1\!/_4\nu_p^2 + \delta_p^2}, \quad E_e = \frac{{}^1\!/_2\nu_p R_e}{{}^1\!/_4\nu_p^2 + \delta_p^2},$$

$$G_{ep} = \frac{p_{ep}\delta_{ep} + q_{ep}{}^1\!/_2\nu_{ep}}{{}^1\!/_4\nu_{ep}^2 + \delta_{ep}^2}, \quad B_{ep} = \frac{{}^1\!/_2\nu_{ep}p_{ep} - q_{ep}\delta_{ep}}{{}^1\!/_4\nu_{ep}^2 + \delta_{ep}^2}, \quad (2.2)$$

where I_0, I_1 and y_0, y_1 are Bessel and Neumann functions of the zeroth and first order, respectively. The parameters λ_j and γ_j are determined from transcendental equations (18) and (19) in [2]. The coefficients A_e, A_p, B_e, B_p, b_e, b_p, C_e, and C_p are obtained if the corresponding subscript is taken into account in (2.2). The coefficients P_{ep}, Q_{ep}, p_{ep}, q_{ep}, R_e, S_e, and S_p, just as the parameters δ_e and δ_p, are defined in [2]. The coefficients in (2.1) and (2.2) are functions of the magnetic-field strength H_0. At high H_0, the coefficients behave as $1/H^{\mu}$, where $\mu \geq 2$.

The obtained expressions for the distributions of the electron and ion concentrations essentially characterize the dynamics of electrostatic plasma polarization in a magnetic field with sudden application of a radial electrostatic field.

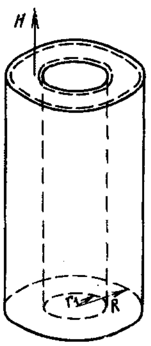
In the absence of a magnetic field, the expressions for the concentrations take the form

$$\begin{aligned} n_e(r, t)|_{H_0=0} = &- n_0 \sum_j \lambda_j M_j(r) \Big\{\frac{eE_0}{m({}^1\!/_4\nu_e^2 + \delta_e^2)} \times \\ &\times \Big(\frac{\omega_p^2\beta_j}{\delta_p({}^1\!/_4\nu_e^2 + \delta_e^2)} - \frac{\alpha_j}{\delta_e}\Big) \times \\ &\times \Big[\delta_e(1 - \exp - {}^1\!/_2\nu_e t\cos\delta_e t) - \frac{\nu_e}{2}\exp - {}^1\!/_2\nu_e t\sin\delta_e t\Big] + \\ &+ \frac{\omega_e^2 eE_0\beta_j}{M({}^1\!/_4\nu_e^2 + \delta_e^2)({}^1\!/_4\nu_p^2 + \delta_p^2)}\Big[(1 - \exp - {}^1\!/_2\nu_p t\cos\delta_p t) - \\ &- \frac{{}^1\!/_2\nu_p}{\delta_p}\exp - {}^1\!/_2\nu_p t\sin\delta_p t\Big]\Big\}, \end{aligned} \quad (2.3)$$

$$\begin{aligned} n_p(r, t)|_{H_0=0} = n_0 \sum_j \gamma_j L_j(r) \Big\{\frac{eE_0}{M({}^1\!/_4\nu_p^2 + \delta_p^2)} \times \\ \times \Big(\frac{\omega_e^2\alpha_j}{{}^1\!/_4\nu_e^2 + \delta_e^2} - \beta_j\Big) \times \\ \times \Big[1 - \exp - {}^1\!/_2\nu_p t\cos\delta_p t - \frac{{}^1\!/_2\nu_p}{\delta_p}\exp - {}^1\!/_2\nu_p t\sin\delta_p t\Big]\Big\}, \end{aligned} \quad (2.4)$$

where ω_{ep}, α_j, and β_j are defined in [2].

It follows from (2.3) and (2.4) that the electron concentration oscillates with the electron as well as with the ion frequencies, wherein the electron-oscillation amplitude includes damped exponential functions in which the damping factor is determined by the frequencies of collisions of electrons and ions with neutral atoms. The ion concentration, however, oscillates only with the ion frequencies, and the damping factor is a function only of the frequency of collisions of ions with neutral atoms.



The application of a magnetic field does not create directional charged-particle motion, but in the presence of an electric field it reduces electron and ion mobility and thereby retards layering of the plasma. As follows from (2.1) and (2.2), the electron and ion concentrations decrease as the magnetic field is increased, and when $H_0 \to \infty$ we have $n_e(r,t) \to 0$ and $n_p(r,t) \to 0$ as $1/H^2$.

When $t \to \infty$, a polarized plasma state is established with the concentration distributions

$$n_e(r, \infty) = -n_0 \sum_j \lambda_j M_j(r) \left\{ A_e + C_e + \frac{S_e}{\nu_e} \right\},$$

$$n_p(r, \infty) = n_0 \sum_j \gamma_j L_j(r) \left\{ A_p + \frac{S_p}{\nu_p} \right\}.$$

3. Since there are no charges in regions 1 and 3 (the walls confining the plasma are impermeable), the potential distribution in these regions satisfies the Laplace equation

$$\Delta U_1 = 0 \quad (r_1 \leqslant r \leqslant r_0),$$
$$\Delta U_3 = 0 \quad (R \leqslant r \leqslant R_1). \tag{3.1}$$

Accordingly, the potential distribution in region 2 satisfies the Poisson equation

$$\Delta U_2 = -4\pi\rho \qquad (r_0 \leqslant r \leqslant R), \qquad \rho = e(n_p - n_e). \tag{3.2}$$

If we substitute the values of n_e and n_p from (2.1) and (2.2) into (3.2), we obtain

$$\Delta U_2 = -4\pi e n_0 \sum_j [\gamma_j L_j(r) \psi_p(t) + \lambda_j M_j(r) \psi_e(t)]. \tag{3.3}$$

As boundary conditions for (3.1), (3.2), and (3.3) we have

$$U_1(r_1, t) = {}^1\!/_2 U_0, \; U_3(R_1, t) = -{}^1\!/_2 U_0, \; U_1(r_0, t) = U_2(r_0, t),$$
$$U_3(R, t) = U_2(R, t),$$
$$[\partial U_1/\partial r]_{r=r_0} = [\partial U_2/\partial r]_{r=r_0},$$
$$[\partial U_3/\partial r]_{r=R} = [\partial U_2/\partial r]_{r=R}. \tag{3.4}$$

The last equation of (3.4) reflects the experimentally observed absence of thin charge layers on the surfaces $r = r_0$ and $r = R$. The functions $\psi_e(t)$ and $\psi_p(t)$ in (3.3) are

$$\psi_e(t) = A_e(1 - \exp - {}^1\!/_2 \nu_e t \cos \delta_e t) -$$
$$-(B_e + b_e t) \exp - {}^1\!/_2 \nu_e t \sin \delta_e t +$$
$$+ C_e(1 - \exp - {}^1\!/_2 \nu_p t) \cos \delta_p t - E_e \exp - {}^1\!/_2 \nu_p t \sin \delta_p t -$$
$$- G_e t \exp - {}^1\!/_2 \nu_e t \cos \delta_e t \frac{S_e}{\nu_e}(1 - \exp - \nu_e t)$$
$$\psi_p(t) = A_p(1 - \exp - {}^1\!/_2 \nu_p t \cos \delta_p t) -$$
$$-(B_p + b_p t) \exp - {}^1\!/_2 \nu_p t \sin \delta_p t -$$
$$- G_p t \exp - {}^1\!/_2 \nu_p t \cos \delta_p t + \frac{S_p}{\nu_p}(1 - \exp - \nu_p t)$$

Equations (3.1), (3.2), and (3.3) completely describe the space and time potential distributions within the plasma capacitor.

If we solve (3.1), (3.2), and (3.3) with allowance for boundary conditions (3.4), we obtain

$$U_1(r, t) = \frac{U_0}{2} \frac{\ln (r^2/r_1) R_1}{\ln (r_1/R_1)} - \frac{4\pi e n_0 \ln (r/r_1)}{\ln (r_1/R_1)} \times$$
$$\times \left\langle \sum_j \left\{ \frac{1}{\gamma_j} [L_j(r_0) - L_j(R)] \psi_p(t) + \right.\right.$$
$$\left.\left. + \frac{1}{\gamma_j} [M_j(r_0) - M_j(R)] \psi_e(t) \right\} \right\rangle, \tag{3.5}$$

$$U_2(r, t) = \frac{U_0}{2} \frac{\ln (r^2/r_1) R_1}{\ln (r_1/R_1)} +$$
$$+ \frac{4\pi e n_0 \ln r_1}{\ln (r_1/R_1)} \left\langle \sum_j \left\{ \frac{1}{\gamma_j} [L_j(r) - L_j(R)] \psi_p(t) + \right.\right.$$
$$\left.\left. + \frac{1}{\gamma_j} [M_j(r) - M_j(R)] \psi_e(t) \right\} \right\rangle - \frac{4\pi e n_0 \ln R_1}{\ln r_1/R_1} \times$$
$$\times \left\langle \sum_j \left\{ \frac{1}{\gamma_j} [L_j(r) - L_j(r_0)] \psi_p(t) + \right.\right.$$
$$\left.\left. + \frac{1}{\lambda_j} [M_j(r) - M_j(r_0)] \psi_e(t) \right\} \right\rangle -$$
$$- \frac{4\pi e n_0 \ln r}{\ln r_1/R_1} \left\langle \sum_j \left\{ \frac{1}{\gamma_j} [L_j(r_0) - L_j(R)] \psi_p(t) + \right.\right.$$
$$\left.\left. + \frac{1}{\lambda_j} [M_j(r_0) - M_j(r)] \psi_e(t) \right\} \right\rangle, \tag{3.6}$$

$$U_3(r, t) = \frac{U_0}{2} \frac{\ln r^2/r_1 R_1}{\ln r_1/R_1} -$$
$$- \frac{4\pi e n_0 \ln (r/R_1)}{\ln (r_1/R_1)} \left\langle \sum_j \left\{ \frac{1}{\gamma_j} [L_j(r_0) - L_j(R)] \psi_p(t) + \right.\right.$$
$$\left.\left. + \frac{1}{\lambda_j} [M_j(r_0) - M_j(R)] \psi_e(t) \right\} \right\rangle. \tag{3.7}$$

The functions $\psi_e(t)$ and $\psi_p(t)$ determine the transient relaxation times and are damped with respect to time.

If we let $t \to \infty$ in (3.5), (3.6), and (3.7), we obtain expressions for the steady potential distribution, which corresponds to transient damping.

It is clear from (3.5), (3.6), and (3.7) that the potential is a continuous function of the two variables r and t. When the potentials U_1, U_2, and U_3 vary with time, a transient current appears in the capacitor circuit. In view of the continuity of the current in regions 1 and 3, which correspond to the potentials U_1 and U_2, the displacement current will be equal to the convection current of region 2. Therefore, the density of the current flowing through the capacitor can be determined as follows:

$$j = \frac{1}{4\pi} \frac{\partial^2 U_1}{\partial r \partial t} \quad \text{or} \quad j = \frac{1}{4\pi} \frac{\partial^2 U_3}{\partial r \partial t}.$$

Therefore,

$$j(r, t) = -\frac{e n_0}{r \ln (r_1/R_1)} \left\langle \sum_j \left\{ \frac{1}{\gamma_j} [L_j(r_0) - L_j(R)] \frac{\partial \psi_p}{\partial t} + \right.\right.$$
$$\left.\left. + \frac{1}{\lambda_j} [M_j(r_0) - M_j(R)] \frac{\partial \psi_e}{\partial t} \right\} \right\rangle.$$

The transient current is damped. After a sufficiently long time interval (much greater than $2/\nu_p$), therefore, the current ceases, where in the charge per unit length of the cylindrical surface is changed by the value

$$\Delta\sigma = 2\pi r_1 \int_0^\infty j(r_1, t)\,dt =$$

$$= -\frac{4\pi e n_0}{2\ln(r_1/R_1)} \left\langle \sum_j \left\{ \frac{1}{\gamma_j}[L_j(r_0) - L_j(R)](A_p + S_p/\nu_p) + \right.\right.$$

$$\left.\left. + \frac{1}{\gamma_j}[M_j(r_0) - M_j(R)]\,\psi_e(t) \right\} \right\rangle. \qquad (3.8)$$

The change in the charge on the plates of the cylindrical capacitor due to the plasma in it involves a change in its capacitance, provided that the potential difference remains constant. This change per unit length of the capacitor can be found from (3.8)

$$\Delta C = \frac{4\pi e n_0}{2U_0 \ln(r_1/R_1)} \sum_j \left\{ \frac{1}{\gamma_j}[L_j(r_0) - L_j(R)]\left(A_p + \frac{S_p}{\nu_p}\right) + \right.$$

$$\left. + \frac{1}{\gamma_j}[M_j(r_0) - M_j(R)]\left(A_e + C_e\frac{S_e}{\nu_e}\right) \right\}. \qquad (3.9)$$

Formula (3.9) makes it possible, from measured ΔC, to determine the current and, therefore, the electron and ion concentrations.

Note that when $H_0 \to \infty$, the capacitance change ΔC, just as $n_e(r, t)$, $n_p(r, t)$, and $j(r, t)$, approaches zero. This means that with very strong magnetic fields ($|H/E| \gg 1$), no charge separation and, therefore, no plasma polarization occur, i.e., the original homogeneous plasma distribution remains unchanged.

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